International Journal of Management, IT & Engineering
Vol. 9 Issue 3, March 2019,
ISSN: 2249-0558 Impact Factor: 7.119
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gate as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Analysis of ε −preopen sets

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Abstract. The notion of preopen set plays a significant role in general topology and hence in this paper, the notion of $\mathcal{E}$ −preopen compactness is established and it is the focal point of this article. Also the relations with other numerous types of compactness are discussed. In addition, new separation axioms are established.

Keywords:$\mathcal{E}$ − preopen set, $\mathcal{E}$ − preopen compact- ness.

Subject Classification. 54C08, 54H05.

Introduction

The notion of pre-open set was introduced by Mashhour et al. [18].Assuming($\mathcal{S}$, φ) be a topological space (or simply, a space) and $\mathcal{B} \subseteq \mathcal{S}$. ThenA is α-open or α set [14, 12] (resp., semi-open [2], semi-closed [2], pre-open [11], preclosed [11], semi-preopen [2], generalized closed [9]) if$\mathcal{B} \subseteq int(cl(int(\mathcal{B})))$ $(resp., \mathcal{B} \subseteq cl(int(\mathcal{B})), int(cl(\mathcal{B})) \subseteq \mathcal{B}, \mathcal{B} \subseteq int(cl(\mathcal{B})), cl(int(\mathcal{B})) \subseteq \mathcal{B}, \mathcal{B} \subseteq cl(int(cl(\mathcal{B}))), cl(\mathcal{B}) \subseteq U$, for every open set U containing $\mathcal{B}$), where $int(\,)$ and $cl(\,)$ are the interior and closure operations, respectively. A is regular-open if $\mathcal{B} = int(cl(\mathcal{B}))$[16]. Complements of regular-open sets are called regular-closed. A is semi-regular [11] if it is both semi-open and semi-closed. $\mathcal{B}$is interior closed [7] if $int(\mathcal{B})$ is semi-closed. $\mathcal{B}$is an $\mathcal{B}$ set [16] if $\mathcal{B} = U \cap C$, where U is an open set and C is a regular closed set. It is known that an A set is semi-open [16]. $\mathcal{B}$is a $\mathcal{L}$set[4] if $\mathcal{B} = U \cap C$ where U is an open set and C is semi-closed. A is locally closed if $\mathcal{B}$ is open in its closure [4]. $\mathcal{B}$is an $\mathcal{B}\mathcal{L}$ − set [4] if $\mathcal{B} = U \cap C$, where U is open and C is semi-regular. Equivalently, $\mathcal{B}$ is an locally closedset if $\mathcal{B} = U \cap C$, where U is open and C is closed. Clearly every $\mathcal{B}$ set is a locally closed set and every locally closed set is a $\mathcal{L}-$ set. Since regularclosed sets are semi-regular and since semi-regular sets are semi-closed, the following implications are obvious:$\mathcal{B}-set \Rightarrow \mathcal{BL}-set \Rightarrow \mathcal{L}-$ set,but none of them of course is reversible [4]. Moreover, since the intersection of an open set and a semi-regular set is always semi-open, every AB set is semi-open.A space ($\mathcal{S}$, φ) is called a partition space if every open subset of $\mathcal{S}$ isclosed, see [4]. ($\mathcal{S}$, φ)is called hyper connected if every open subset of $\mathcal{S}$ is dense. ($\mathcal{S}$, φ)is called extremely disconnected (simply, ED) if every open subset of $\mathcal{S}$has an open closure or equivalently if every regular closed setis open. Spaces that contain two disjoint dense subsets are called resolv- able. ($\mathcal{S}$, φ)is called strongly irresolvable [6] if every open subspace of $\mathcal{S}$ is irresolvable, i.e. itcannot be represented as a disjoint union of twodense subsets. In [5], it has been pointed out that a space is strongly irre- solvable if and only if every preopen set is semi-open. ($\mathcal{S}$, φ) is said to be P-closed [3] (resp., quasi—H—closed (simply, QHC)) if every preopen (resp., open) cover of $\mathcal{S}$ has a finite subfamily the preclosures (resp., closures) of whose members cover $\mathcal{S}$. ($\mathcal{S}$, φ) is said to be strongly compact [10] if everypreopen cover has a finite subcover. ($\mathcal{S}$, φ) is called nearly compact [13] ifevery cover of $\mathcal{S}$ by regular open sets has a finite sub cover. Thompson [15] introduced the class of S-closed spaces, where a space $\mathcal{S}$ is called S-closed if every semi-open cover of $\mathcal{S}$ has a finite subfamily the closures of whose members cover $\mathcal{S}$, or equivalently, if every regular closed cover of $\mathcal{S}$ has afinite subcover. ($\mathcal{S}$, φ) is said to be strongly p-regular [5] (resp., p-regular, almost p- regular) if for each point $x \in \mathcal{S}$ and each pre-closed set (resp., closed set, regular closed set) F such that $x$does not belongs to F, there exist disjoint pre-open sets V and U such that $x\in$ U and $F \subseteq V$.

Throughout this paper, $(\mathcal{S}, \varphi)$ and (Y, δ) stand for topological spaces with no separation axioms assumed unless otherwise mentioned.The fundamental notion of generalized open sets was introduced and explored by several authors. In recent years a number of other generalizations of open sets have been studied. Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. In fact, a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. byutilizing generalized open sets. One ofthe most well known notions and also an inspiration source is the notion of generalized preopen sets introduced in [1]. This class is a superset of the class of semi-closed sets, the class of $\alpha-$sets, the class of $\mathcal{BL}$ sets, the classof A sets and the class of semi-regular sets. Moreover, these investigationslead to solve the problem of finding the continuity dual of some generalized continuous functions in order to have a decomposition of continuity.

Generalized preopen sets in certain spaces

We begin this section by recalling the concept of generalized preopen set and some related results from [1].

Definition 1. [1] A subset $\mathcal{B}$ of a space $\boldsymbol{\mathcal{S}}$ is ε-preopen (simply, $\varepsilon - po$) set if $cl(\mathcal{B}) \subseteq int(U)$, whenever U is a preclosed subset such that int(U)⊇A. Complements of ε −po sets are called generalized preclosed (simply, ε −pc) sets.

By ε −pc($\boldsymbol{\mathcal{S}}$, φ) (resp., $\varepsilon - pc(\boldsymbol{\mathcal{S}}, \varphi))$, we denote the collection of all ε −pc (resp., ε −pc) sets in $\boldsymbol{\mathcal{S}}$.

Lemma 1. [1] A subset $\mathcal{B}$ of a space $\boldsymbol{\mathcal{S}}$ is ε-pc if and only if for every preopen set V contained in $\mathcal{B}, V \subseteq int(\mathcal{B})$.We remark that if $\mathcal{B}$ and $\mathcal{L}$ are ε −po sets, then $\mathcal{B} \cap \mathcal{L}$ need not be a ε −po set, while arbitrary unions of ε −po sets are ε −po sets.

Definition 2. A space ($\boldsymbol{\mathcal{S}}$, φ) is ε −po irresolvable if every preopen set is a ε −po set.

Theorem 1. In a strongly irresolvable space, every semi-open set is a ε −po set.

Theorem 2. If ($\boldsymbol{\mathcal{S}}$, φ) is strongly irresolvable, then it is ε −po-irresolvable.

Proof.Assuming$\mathcal{B}$be a preopen set. Then since $\boldsymbol{\mathcal{S}}$ is strongly irresolvable,$\mathcal{B}$ is semi-open and by Theorem 2, $\mathcal{B}$ is a ε −po- set.
The converse of Theorem 2 is not true in general. $\mathcal{B} = (-\infty, 1]$with the leftray topology on the reals is a ε −po- set and a preopen set but not semi-open.

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The following is a new characterization of open sets.

Theorem 3. [1] A subset $\mathcal{B} \subseteq \mathcal{S}$ is open if and only if $\mathcal{B}$is a ε −po set and a preopen set.

ε −po- compact spaces

Several types of compact spaces were discussed in [3,4,5,8,11]. In this section, ε −po-compact notion is introduced and connections to other several well-known types of compactness are discussed.

Definition 3. A space $(\mathcal{S}, \varphi)$ is ε −po-compact if every ε −po-cover (a cover consisting of ε −po- sets) of $\mathcal{S}$ has a finite subcover.

Equivalently, $(\mathcal{S}, \varphi)$ is ε −po-compact if every ε −po-cover of $\mathcal{S}$ has a finite subcover. A submaximal space is an example of a ε −po-compact space. The proof of the following result follows from the fact that every open set is a ε −po set.

Theorem 4. If $(\mathcal{S}, \varphi)$ is a ε −po-compact space, then it is compact.

Since every compact space is nearly compact and a QHC−space, a ε −po- compact space is nearly compact and QHC.

Theorem 5. If a space $(\mathcal{S}, \varphi)$ is ε −po-irresolvable and ε −po-compact, then it is strongly compact.

Proof.Assuming$\mathcal{B} = \{\mathcal{B}_\alpha : \alpha \in \Delta\}$ be a preopen-cover of $\mathcal{S}$. Then since $\mathcal{S}$ is ε −po-irresolvable, by Theorem 1, A is a ε −po- cover of $\mathcal{S}$ and since $\mathcal{S}$ is ε −po-compact, it has a finite subcover. Thus $\mathcal{S}$ is strongly compact.

The converse of Theorem 4 need not be true since a ε −po- set need not be preopen. In addition, the notions of ε −po-irresolvable and ε-po-compact are independent.

Corollary 1. If a space $(\mathcal{S}, \varphi)$ is ε −po-irresolvable and ε-po-compact, then it is P-closed.

Proof.Assuming$\mathcal{B} = \{\mathcal{B}_\alpha : \alpha \in \Delta\}$ be a preopen cover of $\mathcal{S}$. Since $\mathcal{S}$ is ε −po-irresolvable, $\mathcal{B}_\alpha$is a ε −po set for all $\alpha \in \Delta$ and A is a ε −po-cover of $\mathcal{S}$.Since$\mathcal{S}$is ε −po-compact, it has a finite subcover. Thus $\mathcal{S} \subseteq \cup_{i=1}^{n} \mathcal{B}_{\alpha i}$.But[$\cup_{i=1}^{n} \mathcal{B}_{\alpha i} \subseteq$ [pcl($\mathcal{B}_{\alpha i}$), and so $\mathcal{S}$ is P-closed.Since a ε-poset need not be preopen, a P-closed space need not beε-po-irresolvable or ε −po-compact.

Corollary 2. If ($\mathcal{S}$, φ) is P-closed, T_0 and ε −po-compact, then it is strongly compact.

Proof. By Theorem 3, ($\mathcal{S}$, φ) is strongly irresolvable. By Theorem 2, ($\mathcal{S}$, φ) is ε −po-irresolvable and by Theorem 5, ($\mathcal{S}$, φ) is strongly compact.

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